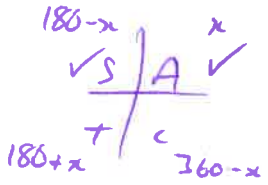
8A

(1) (a) $3\sin x - 1 = 0$

$3\sin x = 1$

$\sin x = \frac{1}{3}$

$\sin^{-1}\left(\frac{1}{3}\right) = 19.47°$

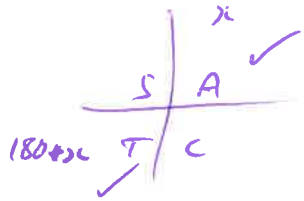


$x = 19.47°, 180 - 19.47$

$x = 19.47°, 160.53°$

(c) $2\tan x = 7$

$\tan x = \frac{7}{2}$



$\tan^{-1}\left(\frac{7}{2}\right) = 74.1°$

$x = 74.1°, 180+74.1°, 360+74.1°$

$x = 74.1°, 254.1°, 434.1°$

8A

2(a) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = 30°$

180 − x	x
✓S	A ✓
T	C

$x = 30, 180 - 30$

$x = 30°, 150°$

2(c) $2\cos t + \sqrt{3} = 0$

$2\cos t = -\sqrt{3}$

$\cos t = -\frac{\sqrt{3}}{2}$

180 − x	
✓S	A
✓T	C

180 + x

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30$

$x = 180 - 30, 180 + 30$

$x = 150°, 210°$

(3) sub in $y = 2$

180 − x ✓	x ✓
S	A
T	C

$2 = 3\sin x + 1$

$1 = 3\sin x$

$\sin x = \frac{1}{3}$

$\sin^{-1}\left(\frac{1}{3}\right) = 19.47°$

$x = 19.47°, 180 - 19.47$

$x = 19.47°, 160.53°$ $\quad (19.47°, 2), (160.53°, 2)$

8B

Look for

1(a) $3\sin 2x - 1 = 0$

$3\sin 2x = 1$

$\sin 2x = \frac{1}{3}$

$180 - x$ ✓	x ✓
S	A
T	C

$\sin^{-1}\left(\frac{1}{3}\right) = 19.47°$

$2x = 19.47°, 180 - 19.47°, \quad (+360°)$

$2x = 19.47°, 160.53°, 379.47°, 520.53°$

$x = 9.74°, 80.3°, 189.74°, 260.3°$

(1)(e) $3\tan(x+60) + 5 = 0$

$3\tan(x+60) = -5$

$\tan(x+60) = -\frac{5}{3}$

$\tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

$180 - x$ ✓
S | A
T | C ✓ $360 - x$

$x + 60 = 180 - 59.04°, 360 - 59.04°$

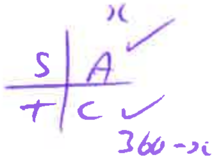
$x + 60 = 120.96°, 300.96°$

$x = 60.96°, 240.96°$

② (b) $\sqrt{2}\cos(x-25) = 1$

$\cos(x-25) = \frac{1}{\sqrt{2}}$

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$



$x - 25 = 45,\ 360 - 45$

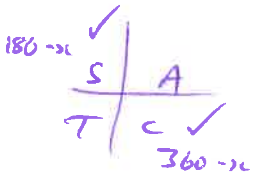
$x - 25 = 45,\ 315°$

$x = 70°,\ 340°$

(d) $\tan(2x - 50) + \sqrt{3} = 0$ $\quad 0 \le x \le 180$

$\tan(2x - 50) = -\sqrt{3}$

$\tan^{-1}(\sqrt{3}) = 60°$



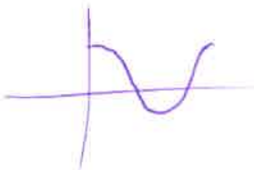
$2x - 50 = 180 - 60,\ 360 - 60,$ (+360)

$2x - 50 = 120,\ 300,\ 420,\ 660$ (+360)

$2x = 170,\ 350,\ 470,\ 710$

$x = 85°,\ 175°,$ $\quad$ (as $0 \le x \le 180$)

(f) $\cos(2x - 45) = 1$ $\quad 0 \le x \le 360$



$\cos^{-1}(1) = 0,\ 360,$

$2x - 45 = 0,\ 360,\ 720$

$2x = 45,\ 405,\ 765$

$x = 22.5°,\ 202.5°,\ \cancel{382.5°}$ $\quad 0 \le x \le 360°$

6B

3(a) $p = 4$, $q = 25°$ (Min normally at $x = 270°$)

(b) $4\sin(x-25) = -1$

$\sin(x-25) = -\frac{1}{4}$

$\sin^{-1}\left(\frac{1}{4}\right) = 14.5°$

S	A
T	C

180+ ✓ 360- ✓

$x - 25 = 180 + 14.5,\ 360 - 14.5°$

$x - 25 = 194.5°,\ 345.5°$

$x = 219.5°,\ 370.5°$

$x = 10.5°,\ 219.5°$

$(10.5°, -1), (219.5°, -1)$

* note 370.5° is out of range but the graph has a wavelength of 360°. Subtract 360 to get a solution in the range $0 \leq x \leq 360$

5(a) Max value occurs when $\sin(n-80) = 1$

$\Rightarrow n - 80 = 90$

$n = 170°$

$D(170) = 12 + 3.7\sin(170-80)$

$= 15.7$ hours of daylight on the 170th day (June 19th).

(b) $14 = 12 + 3.7\sin(n-80)$

$2 = 3.7\sin(n-80)$

$\sin(n-80) = \frac{2}{3.7}$

$\sin^{-1}\left(\frac{2}{3.7}\right) = 32.72$

S ✓	A ✓
T	C

180-

$n - 80 = 32.72,\ 180 - 32.72$

$n - 80 = 32.72, 147.28$

$n = 112.72, 227.28$

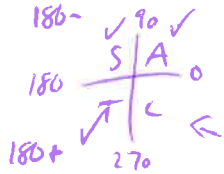
More than 14 hours day light between days 113 and 227.

8C

1(a) $3\tan^2 x = 7$ $\quad 0 \le x \le 270$

$\tan^2 x = \frac{7}{3}$

$\tan x = \pm\sqrt{\frac{7}{3}}$



← not in range

$x = 56.8°, 180-56.8°, 180+56.8°$

$x = 56.8°, 123.2°, 236.8°$

(c) $\sin^2 x - \sin x - 2 = 0$ $\quad 0 \le x \le 360°$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x = 2$ no solutions

$\sin x = -1$ $\quad x = 270°$

(d) $3\cos^2 x - 5\cos x - 2 = 0$ $\quad 0 \le x \le 360$

$(3\cos x + 1)(\cos x - 2) = 0$

$3\cos x + 1 = 0$ $\quad \cos x = 2$ no solutions

$3\cos x = -1$

$\cos x = -\frac{1}{3}$

$\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$

$x = 180-70.5, 180+70.5°$

$x = 109.5°, 250.5°$

8C

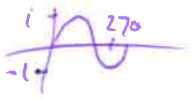
2(a) $5\sin^2(x-30) + 2\sin(x-30) - 3 = 0 \qquad 0° \le x \le 360°$

$(5\sin(x-30) - 3)(\sin(x-30) + 1) = 0$

$5\sin(x-30) - 3 = 0 \quad , \quad \sin(x-30) + 1 = 0$

$5\sin(x-30) = 3 \qquad \sin(x-30) = -1$

$\sin(x-30) = \frac{3}{5} \qquad (x-30) = 270° \qquad x = 300°$



$x - 30 = 36.9, 180 - 36.9$ (180° ✓ S|A ✓ T|C)

$x - 30 = 36.9, 143.1$

$x = 66.9°, 173.1°$

$x = 66.9°, 173.1°, 300°$

(b) $6\cos^2(2x) + 5\cos(2x) - 4 = 0 \qquad 0 \le x \le 360°$

$(3\cos(2x) + 4)(2\cos 2x - 1) = 0$

$3\cos(2x) + 4 = 0 \qquad 2\cos 2x - 1 = 0$

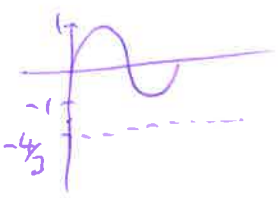
$3\cos 2x = -4 \qquad 2\cos 2x = 1$

$\cos 2x = -\frac{4}{3} \qquad \cos 2x = \frac{1}{2}$ (S|A ✓ T|C ✓ 360−)

no solutions

$2x = 60, 360-60, 60+360, 300+360$

$x = 30°, 150°, 420°, 660°$



8D

1(a) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\pi-$	S ✓	A ✓	
$\pi+$	T	C	$2\pi-$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ rads

(c) $\tan x - \sqrt{3} = 0$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$\pi-$	S	A ✓	
$\pi+$	T ✓	C	2π

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ rads

(e) $2\cos\theta + \sqrt{3} = 0$

$2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\pi+$ ✓	S	A	
$\pi+$ ✓	T	C	$2\pi-$

$\theta = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$ rads

8D

2a) $\sqrt{3}\tan x + 1 = 0$

$\sqrt{3}\tan x = -1$

$\tan x = \frac{-1}{\sqrt{3}}$

$\pi+$ S ✓ | A
$\pi-$ T | C ✓ $2\pi-$

$x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$ rads

2(c) $2\cos x + \sqrt{2} = 0$

$2\cos x = -\sqrt{2}$

$\cos x = -\frac{\sqrt{2}}{2}$

$\pi-$ S ✓ | A
$\pi+$ T ✓ | C

$x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{3\pi}{4}, \frac{5\pi}{4}$ rads

2(e) $5\cos x + 1 = 0$

$5\cos x = -1$

$\cos x = -\frac{1}{5}$

$\pi-$ S ✓ | A
$\pi+$ T ✓ | C

~~$x = 2\pi -$~~

$\cos^{-1}\left(\frac{1}{5}\right) = 1.369$

$x = \pi - 1.369, \pi + 1.369$

$x = 1.78$ rads, 4.511 rads

8D

2(g) $2\cos x - \sqrt{3} = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

S	A ✓
T	C ✓

$2\pi -$

$x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ rads

3(a) $2\sin 2x - 1 = 0$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$\pi -$

S ✓	A ✓
T	C

$2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$+2\pi$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$+2\pi$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ rads

81)

3(c) $2\cos\left(x-\frac{\pi}{6}\right) = -1$

$\cos\left(x-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

S	A
T	C

$\pi - \frac{\pi}{3}$ ✓ , $\pi + \frac{\pi}{3}$ ✓

$x - \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x - \frac{\pi}{6} = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{5\pi}{6}, \frac{9\pi}{6}$ $\left(\frac{2\pi}{3} + \frac{\pi}{6} = \frac{4\pi}{6} + \frac{\pi}{6} = \frac{5\pi}{6}\right)$

$x = \frac{5\pi}{6}, \frac{3\pi}{2}$

3(e) $3\tan(3x) + 3 = 0 \quad 0 \le x \le \pi$

$3\tan(3x) = -3$

$\tan(3x) = -1$

$\tan^{-1}(1) = \frac{\pi}{4}$

S ✓	A
T	C ✓

$\pi - \frac{\pi}{4}$, $2\pi - \frac{\pi}{4}$, ...

$3x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}$ (+2π, +2π)

$x = \frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}$ out of range.

$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ rads

8D

3(g) $\sqrt{2}\cos\left(x-\frac{\pi}{3}\right)=1$

$\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$

S	A ✓
T	C ✓

$2\pi -$

$x-\frac{\pi}{3}=\frac{\pi}{4}, 2\pi-\frac{\pi}{4}$

$x-\frac{\pi}{3}=\frac{\pi}{4}, \frac{7\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{3}, \frac{7\pi}{4}+\frac{\pi}{3}$

$x=\frac{3\pi}{12}+\frac{4\pi}{12}, \frac{21\pi}{12}+\frac{4\pi}{12}$

$x=\frac{7\pi}{12}, \frac{25\pi}{12}$ ← out of range, subtract 2π to get value in range $\left(\frac{25\pi}{12}-2\pi=\frac{\pi}{12}\right)$

$x=\frac{\pi}{12}, \frac{7\pi}{12}$

8D

Q4

$y = y$

$2\cos 2x + 3 = 4$

$2\cos 2x = 1$

$\cos 2x = \frac{1}{2}$

S	A ✓
T	C ✓

$2\pi -$

$2x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \ldots$

$2x = \frac{\pi}{3}, \overset{+2\pi}{\frac{5\pi}{3}}, \underset{+2\pi}{\frac{7\pi}{3}}, \frac{11\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

coordinate $\left(\frac{\pi}{6}, 4\right), \left(\frac{5\pi}{6}, 4\right), \left(\frac{7\pi}{6}, 4\right), \left(\frac{11\pi}{6}, 4\right)$

81)

Q6

$$N = 60 - 50\cos\left(\frac{2\pi}{11}t\right)$$

$$100 = 60 - 50\cos\left(\frac{2\pi}{11}t\right)$$

$$-40 = -50\cos\left(\frac{2\pi}{11}t\right)$$

$$-\frac{40}{50} = \cos\left(\frac{2\pi}{11}t\right)$$

$\pi -$ ✓ S | A
$\pi +$ ✓ T | C

$$\cos^{-1}\left(\frac{40}{50}\right) = 0.6435...$$

$$\frac{2\pi}{11}t = \pi - 0.6435..., \pi + 0.6435$$

$$\frac{2\pi}{11}t = 2.498..., 3.785...$$

$$t = 4.373..., 6.626...$$

t = Year 4 and Year 7

$\left(\frac{2\pi}{11}t = 2.495\right.$

$2\pi t = 2.495 \times 11$

$t = \frac{2.495 \times 11}{2\pi}$

8E

1a) $2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$\pi -$ S | A
$\pi +$ T | C $2\pi -$

$x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

1(b) $\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$\pi -$ S | A
$\pi +$ T | C $2\pi -$

$x = \frac{\pi}{3}, \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

8E

2(a) $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

$2\cos x = 1$ $\quad$ $\cos x = -1$

$\cos x = \frac{1}{2}$ $\quad$ $x = \pi$

$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

S | A
T | C, $2\pi -$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

(b) $2\sin^2 x - \sqrt{3}\sin x = 0$

$\sin x(2\sin x - \sqrt{3}) = 0$

$\sin x = 0$ $\quad$ $2\sin x - \sqrt{3} = 0$

$x =$  $\quad$ $2\sin x = \sqrt{3}$

$x = 0, \pi, 2\pi$ $\quad$ $\sin x = \frac{\sqrt{3}}{2}$

$\pi -$ S | A
T | C

$x = \frac{\pi}{3}, \pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, 2\pi$

8E

2c $6\sin^2 x - \cos x - 1 = 0$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$6(1-\cos^2 x) - \cos x - 1 = 0$

$6 - 6\cos^2 x - \cos x - 1 = 0$

$-6\cos^2 x - \cos x + 5 = 0$

$0 = 6\cos^2 x + \cos x - 5$

$0 = (6\cos x - 5)(\cos x + 1)$

$6\cos x - 5 = 0$ $\quad\quad$ $\cos x + 1 = 0$

$6\cos x = 5$ $\quad\quad$ $\cos x = -1$

$\cos x = 5/6$ $\quad\quad$ $x = \pi$

$x = 0.5856\ldots, 2\pi - 0.5856$

$x = 0.59, 5.70$ rads

8F

choose $\cos 2x = 2\cos^2 x - 1$

2(a) $\cos 2x + 3\cos x + 2 = 0$

$2\cos^2 x - 1 + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$ $\quad$ $\cos x + 1 = 0$

$2\cos x = -1$ $\quad$ $\cos x = -1$

S | A
T | C

$\cos x = -\frac{1}{2}$ $\quad$ $x = 180°$

$\cos^{-1}\left(\frac{1}{2}\right) = 60°$

$x = 180 - 60,\ 180 + 60$

$x = 120°,\ 240°$

$x = 120°,\ 180°,\ 240°$

8F

use $\cos 2p = 1 - 2\sin^2 p$

2. $\cos 2p - 7\sin p - 4 = 0$

$1 - 2\sin^2 p - 7\sin p - 4 = 0$

$-2\sin^2 p - 7\sin p - 3 = 0$

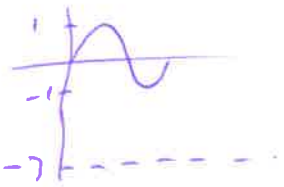
$0 = 2\sin^2 p + 7\sin p + 3$

$0 = (2\sin p + 1)(\sin p + 3)$

$2\sin p + 1 = 0$ $\quad$ $\sin p + 3 = 0$

$2\sin p = -1$ $\quad$ $\sin p = -3$

$\sin p = -\frac{1}{2}$ $\quad$ no solutions



S | A
T | C

180+ 360-

$\sin^{-1}(\frac{1}{2}) = 30°$

$p = 180+30,\ 360-30$

$p = 210°, 330°$

8F

2e $5\cos 2x - \cos x + 2 = 0$

use $\cos 2x = 2\cos^2 x - 1$

$5(2\cos^2 x - 1) - \cos x + 2 = 0$

$10\cos^2 x - 5 - \cos x + 2 = 0$

$10\cos^2 x - \cos x - 3 = 0$

$(5\cos x - 3)(2\cos x + 1) = 0$

$5\cos x - 3 = 0$ $\quad$ $2\cos x + 1 = 0$

$5\cos x = 3$ $\quad$ $2\cos x = -1$

$\cos x = \frac{3}{5}$ $\quad$ $\cos x = -\frac{1}{2}$

S | A
T | C

$x = 53.1, 360 - 53.1$ $\quad$ $x = 180 - 60, 180 + 60$

$x = 53.1°, 120°, 240°, 306.9°$

8F

use $\cos 2x = 2\cos^2 x - 1$

2g $2\cos 2x + \cos x - 1 = 0$

$2(2\cos^2 x - 1) + \cos x - 1 = 0$

$4\cos^2 x - 2 + \cos x - 1 = 0$

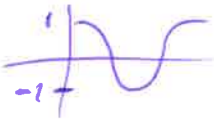
$4\cos^2 x + \cos x - 3 = 0$

$(4\cos x - 3)(\cos x + 1) = 0$

$4\cos x - 3 = 0$ $\quad$ $\cos x + 1 = 0$

$4\cos x = 3$ $\quad$ $\cos x = -1$

$\cos x = 3/4$ $\quad$ $x = 180°$



S | A ✓
T | C ✓ 360 -

$x = 41.4°, 360 - 41.4°$

$x = 41.4°, 180°, 318.6°$

8F

3(a) $\sin 2x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$

$\cos x (2\sin x + 1) = 0$

~~Or~~ $\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$x = 90, 270$ $\quad$ $2\sin x = -1$

$\sin x = -\frac{1}{2}$

180-S | A
180+T | C ✓
360

$\sin^{-1}(\frac{1}{2}) = 30$

$x = 360-30, 180+30$

$x = 330, 210$

~~$x = 90, 150, 210, 270$~~

$x = 90, 210, 270, 330$

8F

3(b) $\sin 2x - 3\sin x = 0$

$2\sin x \cos x - 3\sin x = 0$

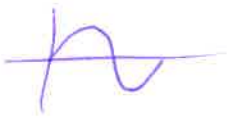
$\sin x (2\sin x - 3) = 0$

$\sin x = 0$ $\quad$ $2\sin x - 3 = 0$

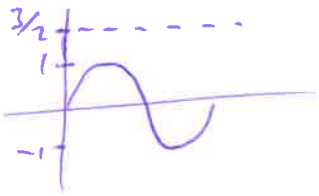
$2\sin x = 3$

$\sin x = \frac{3}{2}$

no solutions



$x = 0, 180, 360$



3(c) $3\sin 2t - 2\sin t = 0$ $\quad$ $0 \le x \le 720$

$3 \times 2\sin t \cos t - 2\sin t = 0$

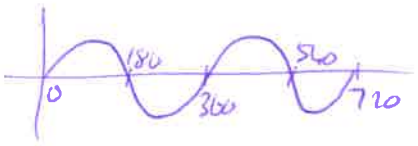
$6\sin t \cos t - 2\sin t = 0$

$2\sin t (3\cos t - 1) = 0$

$2\sin t = 0$ $\quad$ $3\cos t - 1 = 0$

$3\cos t = 1$

$\cos t = \frac{1}{3}$



S	A ✓
T	C ✓ 360-

$t = 0, 180, 360, 540, 720$

$t = 70.53, 360 - 70.53, \ldots$

$+360$

$t = 70.5, 289.5, 430.5, 649.5$

$+360$

$t = 0, 70.5, 180, 289.5, 360, 430.5, 540, 649.5, 720$

8F

~~3d~~

3(d) $5\sin 2t = 3\cos t$

$5 \times 2\sin t\cos t = 3\cos t$

$10\sin t\cos t - 3\cos t = 0$

$\cos t(10\sin t - 3) = 0$

$\cos t = 0$ $\qquad$ $10\sin t - 3 = 0$

$10\sin t = 3$

$\sin t = \frac{3}{10}$

$t = 90°, 270°$

S ✓	A ✓
T	C

$t = 17.5, 180 - 17.5°$

$t = 17.5, 162.5°$

$t = 17.5°, 90°, 162.5°, 270°$

8F

(5) $y = y$

$2\cos 2x - 1 = 3\sin x$

use $\cos 2x = 1 - 2\sin^2 x$

$2\cos 2x - 3\sin x - 1 = 0$

$2(1 - 2\sin^2 x) - 3\sin x - 1 = 0$

$2 - 4\sin^2 x - 3\sin x - 1 = 0$

$-4\sin^2 x - 3\sin x + 1 = 0$

$0 = 4\sin^2 x + 3\sin x - 1$

$0 = (4\sin x - 1)(\sin x + 1)$

$4\sin x - 1 = 0$

$4\sin x = 1$

$\sin x = \frac{1}{4}$ ✓ ✓

180- S A T C

$x = 14.5°, 165.5°$

$\sin x + 1 = 0$

$\sin x = -1$

$x = 270°$

$x = 14.5°, 165.5°, 270°$

8F

(6) (a) $p = 2$ (altitude = 2)

$q = 2$ (2 waves in 360°)

(b) $y = y$

$2\sin 2x = \sin x$

$2 \times 2\sin x \cos x = \sin x$

$4\sin x \cos x - \sin x = 0$

$\sin x (4\cos x - 1) = 0$

$\sin x = 0$ $\quad$ $4\cos x - 1 = 0$

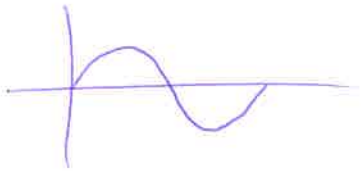
$4\cos x = 1$

$\cos x = 1/4$

$x = 75.5, 360 - 75.5$

S	A ✓
T	C ✓

360 -



$x = 0°, 180°, 360°$

$x = 0, 75.5°, 180, 284.5°, 360°$

8G

use $\cos 2x = 2\cos^2 x - 1$

2(a) $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

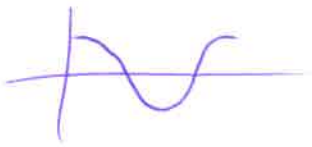
$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\qquad$ $\cos x + 1 = 0$

$2\cos x = 1$ $\qquad$ $\cos x = -1$

$\cos x = \frac{1}{2}$ $\qquad$ $x = \pi$



S | A ✓

T | C ✓

$2\pi -$

$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

8G

2(c)

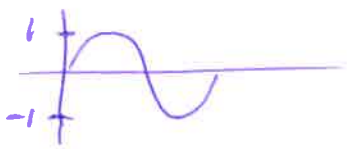
$\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

$\sin x (2\cos x - 1) = 0$

$\sin x = 0$



~~$x = 0, 180, 360$~~

$x = 0, \pi, 2\pi$

$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

S	A ✓
T	C ✓ 2π

$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ rads

8G

both $2x$ so use $\cos^2 2x + \sin^2 2x = 1$, $\cos^2 2x = 1 - \sin^2 2x$

3 (a) $2\cos^2 2x - \sin 2x - 1 = 0$

$2(1 - \sin^2 2x) - \sin 2x - 1 = 0$

$2 - 2\sin^2 2x - \sin 2x - 1 = 0$

$-2\sin^2 2x - \sin 2x + 1 = 0$

$0 = 2\sin^2 2x + \sin 2x - 1$

$0 = (2\sin 2x - 1)(\sin 2x + 1)$

$2\sin 2x - 1 = 0$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

180 ✓ ✓
$\pi - \frac{S|A}{T|C}$

~~$2x = 30, 180-30, \ldots$~~

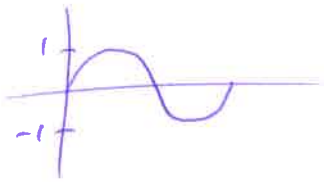
$2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \ldots$

$2x = \frac{\pi}{6}, \overset{+2\pi}{\frac{5\pi}{6}}, \frac{13\pi}{6}, \underset{+2\pi}{\frac{17\pi}{6}}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$\sin 2x + 1 = 0$

$\sin 2x = -1$



$2x = \frac{3\pi}{2}, \overset{+2\pi}{\frac{7\pi}{2}}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$

8H

1(a) $7\cos x + 4\sin x = K\cos(x-\alpha)$

$= K(\cos x \cos\alpha + \sin x \sin\alpha)$

$7\cos x + 4\sin x = K\cos\alpha\cos x + K\sin\alpha \sin x$

$K\cos\alpha = 7$

$K\sin\alpha = 4$

α: $\tan\alpha = \frac{K\sin\alpha}{K\cos\alpha} = \frac{4}{7}$

$\alpha = \tan^{-1}\left(\frac{4}{7}\right)$ ✓ ✓ S A T C ✓

$\alpha = 29.7°$

K: $K = \sqrt{7^2+4^2}$

$K = \sqrt{65}$

$7\cos x + 4\sin x = \sqrt{65}\cos(x-29.7)°$

1(b) $7\cos x + 4\sin x = 5$

$\sqrt{65}\cos(x - 29.74)^\circ = 5$

$\cos(x - 29.74)^\circ = \frac{5}{\sqrt{65}}$

S	A ✓
T	C ✓

360 -

$x - 29.74 = 51.67, 360 - 51.67$

$x - 29.74 = 51.7, 308.31$

$x = 81.4^\circ, 338.01^\circ$

8H

$3\cos x + \sin x = 1$

Let $3\cos x + \sin x = K\sin(x+\alpha)$

$= K\sin x\cos\alpha + K\cos x\sin\alpha$

$3\cos x + \sin x = K\cos\alpha \sin x + K\sin\alpha \cos x$

$K\sin\alpha = 3$

$K\cos\alpha = 1$

α: $\tan\alpha = \frac{K\sin\alpha}{K\cos\alpha} = \frac{3}{1}$

S A ✓
T C ✓

$\alpha = \tan^{-1}(3)$

$\alpha = 71.565\ldots$

K: $K = \sqrt{3^2+1^2}$

$K = \sqrt{10}$

$3\cos x + \sin x = \sqrt{10}\sin(x+71.565)^\circ$

$3\cos x + \sin x = 1$

$\sqrt{10}\sin(x+71.565) = 1$

$\sin(x+71.565) = \frac{1}{\sqrt{10}}$

$x + 71.565 = 18.435, 180 - 18.435$ ✓ ✓

180 - S A T C

$x + 71.565 = 18.435, 161.565$

$x = -53.13, 90°$

↖ out of range. Add on 360° to get solution in range.

$x = 90°, 306.9°$

8H

4(e) $2\cos x + 3\sin x - 1 = 0$

$2\cos x + 3\sin x = 1$

Let $2\cos x + 3\sin x = k\cos(x+\alpha)$

$= k(\cos x \cos\alpha - \sin x \sin\alpha)$

$= k\cos\alpha \cos x - k\sin\alpha \sin x$

$k\cos\alpha = 2$

$-k\sin\alpha = 3$

$k\sin\alpha = -3$

α: $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha} = -\frac{3}{2}$

S | A ✓

T | C ✓ ✓

$\tan^{-1}\left(\frac{3}{2}\right) = 56.310$

$\alpha = 360 - 56.310$

$= 303.690$

k: $k = \sqrt{2^2 + (-3)^2}$

$k = \sqrt{13}$

$2\cos x + 3\sin x = \sqrt{13}\cos(x + 303.69)^\circ$

$2\cos x + 3\sin x = 1$

$\sqrt{13}\cos(x + 303.69) = 1$

8H $\cos(x + 303.69) = \frac{1}{\sqrt{13}}$

S | A ✓
T | C ✓
360 -

$x + 303.69 = 73.90, 360 - 73.90$

$x + 303.69 = 73.90°, 286.10°$

$x = -229.79°, -17.59°$

both values out of range so add 360 to each value to bring in range

$x = 130.21°, 342.41°$

8I

1(a) $2\cos x \cos\frac{\pi}{3} - 2\sin x \sin\frac{\pi}{3} = 1$

$2(\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}) = 1$

$2\cos(x+\frac{\pi}{3}) = 1$

$\cos(x+\frac{\pi}{3}) = \frac{1}{2}$

S ✓	A
T	C ✓

$2\pi -$

$(x+\frac{\pi}{3}) = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$x+\frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = 0, \frac{4\pi}{3}$

($0 \le x < 2\pi$ so 2π not in range)

1(c) $5\cos 2x \cos 20 + 5\sin 2x \sin 20 + 3 = 0$

$5(\cos 2x \cos 20 + \sin 2x \sin 20) + 3 = 0$

$5\cos(2x - 20) = -3$

$\cos(2x-20) = -\frac{3}{5}$ $\quad 0 \le x \le 270$

$\cos^{-1}(\frac{3}{5}) = 53.13°$

180 − ✓ S	A
180 + ✓ T	C

$+360$

$2x - 20 = 126.87°, 233.13, 486.87, 593.13$

$+360°$

$2x = 146.87°, 253.13°, 506.87°, 613.13$

$x = 73.4°, 126.6°, 253.4°,$ ~~306.6~~ out of range

8I

2(a) $(\cos x - \sin x)(\cos x + \sin x)$

$= \cos^2 x + \cos x \sin x - \cos x \sin x - \sin^2 x$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

2(b) $(\cos x - \sin x)(\cos x + \sin x) + 1 = 0$

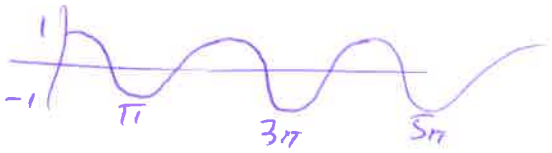
$\cos 2x + 1 = 0$

$\cos 2x = -1$

$2x = \pi, 3\pi, 5\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$0 \leq x \leq 3\pi$



8I

3(i) $\sin 3x$

$= \sin(2x + x)$ $\quad \cos 2x = 1 - 2\sin^2 x$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x (1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= \underline{3\sin x - 4\sin^3 x}$

(b) $3\sin x - 4\sin^3 x = \frac{1}{2}$

$\sin 3x = \frac{1}{2}$ $\quad \pi - \frac{S \mid A}{T \mid C}$ ✓ ✓

$3x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \ldots$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ $\quad +2\pi$ $\quad +2\pi$

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$